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Elevation determination of nunataks in the Grove Mountains

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Abstract A majority of the exposed nunataks located in the Grove Mountains of the Antarctic interior have yet to have had their elevations measured. The elevations of Mason Peak and Wilson Ridge were precisely determined by the Grove Team of the 26th CHINARE in 2010, with Mason Peak turning out to be the highest of the Grove Mountains. Considering that both Mason Peak and Wilson Ridge are difficult to climb because of their cragginess, we first selected three control points on the ice surface near Mason Peak and positioned them with GPS. Thus, accurate elevations of Mason Peak and Wilson Ridge could be calculated from three directions using forward intersection and trigonometric leveling of a high-precision theodolite at the chosen control points. The results provide basic geodetic information that can be referred to as high-precision control points for surveying and mapping in this part of Antarctica. This paper elaborates on the process of measurement and computation of the mountains summit elevations, and also analyzes the details of the principal elements influencing the accuracy of trigonometric leveling, the determination of refraction coefficients k, and observations of structure and distance.

Keywords **Grove Mountains, Mason Peak, Wilson Ridge, trigonometric leveling, GPS, forward intersection**

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0 Introduction

Mason Peak and Wilson Ridge are among the highest of the 64 exposed nunataks located in ice and snow fields about 2 000 m asl in the Grove Mountains, in the hinterland of East Antarctica. The relative heights of these nunataks, and in particular the determination of the highest, had not been verified by previous surveys in the area. Through precise surveying and measuring methods, the Grove Mountain Team of the 26th CHINARE (Chinese National Antarctic Research Expedition) in 2009/2010 successfully determined the elevation of the summit of Mason Peak (2 362.9 m asl) showing that it is the highest nunatak in the Grove Mountains, whereas the maximum elevation of Wilson Ridge is 2 325.1 m asl.

During four previous CHINARE surveys in the Grove Mountains, surveyors have carried out fieldwork on ice surface topography and numerous exposed nunataks. Due to access difficulties caused by rugged topography and harsh weather conditions, the elevations of Mason Peak and Wilson Ridge had not been precisely determined prior to this work. Despite preliminary estimates provided by surveys decades ago, the elevation of Mason Peak based on satellite imagery had a large relative error, e.g., a 200 m elevation gap exists between the two maps published by the Australian Antarctic Division.

Considering the difficulty of climbing Mason Peak and Wilson Ridge, we first selected three control points on the ice surface near Mason Peak and positioned them with GPS. Thus, accurate elevations of Mason Peak and

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Wilson Ridge could be calculated from three directions using forward intersection and trigonometric leveling by setting a high-precision theodolite on the chosen control points.

1 Control point measurements and calculations

To precisely determine the elevation and plane position of Mason Peak, we first set up a highly-accurate control point MS1 at the foot of Mason Peak using comprehensive GPS measurements over a 12-hour continuous observation period. Then, based on MS1, two more GPS control points MS2 and MS3 were established using GPS RTK (Real Time Kinematic) (Figure 1).

The precise coordinates of MS1 were calculated with GAMIT/GLOBK. To do this, we chose two known IGS sites near Zhongshan Station, e.g., DAV1 (Davis Station) and MAW1 (Mawson Station), and Zhongshan Station as a tracking station ZHON. We then used these three sites as origin points to calculate the coordinates of MS1.

The coordinates of the three known stations above are based on ITRF2000, 2010.001. The epoch information is shown in Table 1.

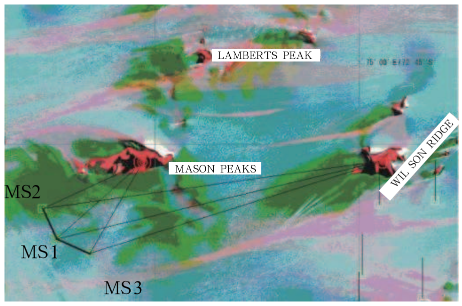

Figure 1 Sketch map of elevation determination of nunataks in Grove Mountains.

The coordinates of the two IGS stations were acquired from the SOPAC website[1]. The coordinates of Zhongshan Station were calculated based on DAV1, MAW1 and SYOG, and after adjustment with the global network of IGS stations, the coordinates for the first day of the year 2010 were obtained.

Using DAV1, MAW1 and ZHON as control points, we obtained a baseline for MS1 and then referenced its coordinates to WGS84. The precision of the GPS baseline, coordinates and precision of MS1 is shown in Tables 2 and 3.

Table 1 Known stations and coordinates (ITRF2000, 2010.001 Epoch)

Site	Station style	X	Y	Z
DAV1	IGS station	486 854.566	2 285 099.237	–5 914 955.733
MAW1	IGS station	1 111 287.188	2 168 911.229	–5 874 493.633
ZHON	GPS tracking station	531 375.205	2 190 320.319	–5 946 671.112

Table 2 Precision of GPS baseline

Baseline	Baseline length L/m	Root mean square error σ/m
MS1-ZHON	391 203.832 0	0.003 1

Table 3 Coordinate and precision of MS1

Site	$(X/\sigma x)$/m	$(Y/\sigma y)$/m	$(Z/\sigma z)$/m
MS1	503 359.259	1 821 231.114	-6 073 275.652
	0.007	0.007	0.016

2 Determining the plane position of the nunataks using forward intersection

To measure specific elevations, we first determined the plane position of the two nunataks. We set up theodolites at stations MS1, MS2, MS3 to measure the horizontal angle of Mason Peak and Wilson Ridge. This permitted the determination of the forward intersection positions of the theodolites and precisely measured the plane positions of the two mountain peaks. We used a WILD T2 theodolite to measure the angles; two observation sets were required, which resulted in angle measurements with an accuracy of <2″.

The formulas for forward intersection are as follows (Figure 2)[2].

$$x_P = \frac{x_A \cot\beta + x_B \cot\alpha - y_A + y_B}{\cot\alpha + \cot\beta}$$

$$y_P = \frac{y_A \cot\beta + y_B \cot\alpha + x_A - x_B}{\cot\alpha + \cot\beta}$$

For easy calculation using the formulas above, we purposely projected the geodetic coordinates of MS1, MS2, MS3 to a Gaussian coordinate system. To minimize deformation in the survey area by projection, we

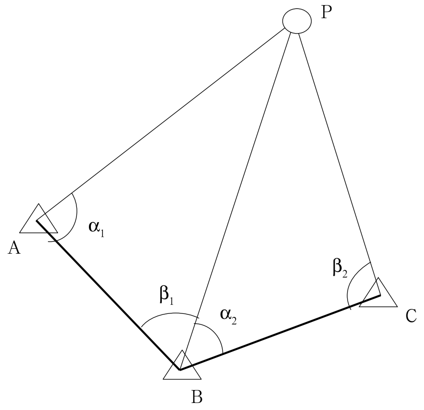

Figure 2 Sketch map of angular forward intersection.

chose the mean longitude as the central meridian L_0=75° and the mean elevation as 2 100 m (set as the as elevation datum for the projection). The precise plane position and accuracy index are shown in Table 4.

The accuracy of the plane position obtained by forward intersection for Mason Peak is better than Wilson Ridge due to the differences in the observation structure for the two mountains. For Mason Peak, the observing stations of MS1, MS2, and MS3 were good enough for positioning during intersection, whereas the distance from the control points to Wilson Ridge is too long for an accurate positioning. In addition, the top of Wilson Ridge lacks undulatory features, which makes it difficult to align and the aiming accuracy that is relatively low.

Table 4 Coordinates and precision of Mason Peak and Wilson Ridge

Target	Latitude	Accuracy/m	Longitude	Accuracy/m
Mason Peak	–72°48′59.76″	±0.219	74°40′40.41″	±0.189
Wilson Ridge	–72°48′51.22″	±1.808	75°02′35.73″	±0.450

3 Measuring the height of a nunatak summit using trigonometric leveling

3.1 Basic formulas for trigonometric leveling

The basic principles and formulas for calculating height differences by trigonometric leveling have been discussed in detail in surveying reports[3].

In Figure 3, suppose s_0 is the observational horizontal distance between points A and B. The instrument is put on point A, and the instrument elevation is determined to be i_1. Point B is the point used for alignment and the target elevation is v_2; R is the radius of A′B′ on the reference ellipsoid. PE and PF are the geoids that pass through points P and A, respectively. $\overline{PC}$ is the tangent line to PE that passes point P, and PN is the curve of the optical path. Because of the effect of atmospheric refraction, when the telescope at point P aims in the direction of PM (i.e., tangent to PN), the rays projected from point N arrive just at the horizontal line of the telescope. This means that when the instrument is put on point A, the vertical angle between points P and M is $\alpha_{1,2}$.

Figure 3 shows that the height difference between points A and B is:

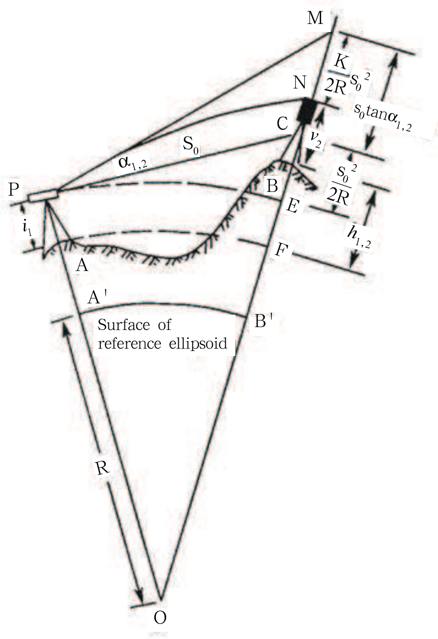

Figure 3 Principles of trigonometry leveling.

$$h_{1,2} = BF = MC + CE + EF - MN - NB \qquad (1)$$

where EF is the instrument elevation i_1, NB is the target elevation v_2, and CE and MN are the earth curvature and the effect of atmospheric refraction.

$$CE = \frac{1}{2R}s_0^2, \qquad MN = \frac{1}{2R'}s_0^2$$

where R' is the radius of optical path $\widehat{PN}$ at point N. If we let $\frac{R}{R'} = K$, then:

$$MN = \frac{1}{2R'} \cdot \frac{R}{R} S_0^2 = \frac{K}{2R} S_0^2$$

where k is defined as the refraction coefficient.

Due to the extremely low ratio of the horizontal distance between points A and B, s_0 to radius R, PC is supposed to be approximately vertical to OM, i.e., $PCM \cong 90°$. Thus, ΔPCM is regarded as a right triangle. Then MC in formula (1) is:

$$MC = s_0 \tan\alpha_{1,2}$$

and the height difference between A and B is:

$$\begin{aligned} h_{1,2} &= s_0 \tan\alpha_{1,2} + \frac{1}{2R} s_0^2 + i_1 - \frac{K}{2R} s_0^2 - v_2 \\ &= s_0 \tan\alpha_{1,2} + \frac{1-K}{2R} s_0^2 + i_1 - v_2 \end{aligned} \quad (2)$$

From the formula above we conclude that the elements that influence the accuracy of trigonometric leveling are mainly the observational accuracy of the vertical angle and the verifying accuracy of the refraction coefficient k.

3.2 Determining the atmospheric refraction coefficient *k*

Vertical refraction arises when rays pass through an atmosphere with variable density. The determining factor for the gradient of atmosphere density is the gradient of atmospheric temperature, τ. Thus dealing with the vertical refraction problem, especially measuring the gradient of atmospheric temperature, is of great importance in obtaining acceptable trigonometric leveling results. The atmospheric refraction coefficient k varies according to area, climate, season, ground cover and the height above the ground. At present, it is impossible to precisely measure the value of k. Experiments that have been carried out in China show that the value of k is the lowest and relatively steady about noon[5]; the value of k is relatively high and changes quickly during sunrise and sunset. Therefore, the best time for observing vertical angles is 10:00 to 16:00 local time when the value of K recorded is generally 0.08–0.14. However, the situation at Antarctica, where the surface is dominated by ice and snow, is different. The snow surface absorbs little heat, the polar day lasts for many months, and the katabatic winds that occur in the polar regions are extremely strong. Together, these elements result in an atmosphere that is even and steady, which facilitates the measurement of vertical angles all day long.

According to the principles of atmospheric physics and combining them with abundant statistics of meteorological observation, Brocks[4] produced a formula for calculating the refraction coefficient k:

$$k = 6.706 \frac{P}{T^2}(3.42 + \tau)\left[1 - \frac{1}{3}(3.42 + 2\tau)\frac{\Delta H}{T}\right]\sin Z \quad (3)$$

where P is atmospheric pressure (units: mmHg), T is absolute temperature (K) at the ground station, τ is the vertical gradient of temperature [units: °C·(100 m)$^{-1}$], ΔH is the height difference between a low observation station and a high observation station (units: 100 m), and Z is the zenith distance between the two stations. Brocks' formula, derived for the situation where the rays are through a "free" atmosphere, works for the calculation of k when the majority of the observation rays exceed the ground surface in a relatively large value. This formula was applied to measure the elevation of Mount Qomolangma many times and has been proven reliable[5−6]. It also works for the Grove Mountains with a mean elevation of over 2 000 m.

Due to the limitations of the research conditions, we did not manage to release balloons to measure the temperature gradient during this expedition to the Grove Mountains. So, to calculate refraction coefficients, we applied the vertical gradient of mean temperature in the troposphere:

$$\tau = -0.65°\text{C}/(100 \text{ m}) \quad (4)$$

By measuring atmospheric pressure and temperature, and by observing zenith distance, using formulas (3) and (4), we calculated a refraction coefficient for the Grove Mountains:

$$k = 0.15 \quad (5)$$

Consider that the distances from MS1, MS2 and MS3 to Mason Peak are 5.1–5.6 km, and a 0.01 error in k, leads to a 0.02 m height error in the Mason Peak elevation. Therefore, the effect of the refraction coefficient at Mason Peak can be neglected.

The distances from MS1, MS2 and MS3 to Wilson Ridge are comparatively longer (i.e., about 15.4–17.1 km), and due to a 0.01 error in k, this leads to a 0.2 m height error in the Wilson Ridge elevation. The effect of the refraction coefficient on Wilson Ridge is too large to be neglected.

3.3 Determination of the Mason Peak and Wilson Ridge summit elevations

As described above, WILD T2 theodolite stations at MS1, MS2, and MS3 near Mason Peak were used to determine the precise elevation of Mason Peak and Wilson Ridge using trigonometric leveling. We carried out two observation sets for the angles resulting in an accuracy of angle measuring of $\pm 2''$.

An error of $\pm 2''$ for an angle measurement can introduce a ± 0.05 m height error to the Mason Peak elevation and ± 0.16 m to that of Wilson Ridge.

After putting the observed values of vertical angles in Table 5 into formula (2), we calculated the elevations of Mason Peak and Wilson Ridge. Comprehensively considering the observational error and atmosphere vertical refraction error, and after the application of appropriate adjustments for redundant measurements, we derive an evaluation of precision (Table 6).

Table 5 Results of vertical angle observation

Target	Site		
	MS1	MS2	MS3
Mason Peak	5°48′18″	6°09′51″	6°09′06″
Wilson Ridge	1°45′59″	1°45′19″	1°52′00″

Table 6 Elevation results of Mason Peak and Wilson Ridge

Target Geoid	elevation/m	Precision/m
Mason Peak	2 362.869	±0.179
Wilson Ridge	2 325.078	±0.418

4 Conclusion

We have precisely determined the geoid elevation in the area of Mason Peak and Wilson Ridge by combining GPS and trigonometric leveling using theodolite measuring angles. This has solved a local puzzle of geodetic heights. The result provides information that can be referred to as highly-precise control points for Antarctic surveying and mapping.

The geoid elevation of the summit of the Mason Peak is 2 363 m asl. It is the highest peak in the Grove Mountains, 20 m higher than Mount Harding in the core area of the Grove Mountains[7].

To improve the accuracy of elevation determinations, we set instruments on three chosen control points to observe the vertical angles in Mason Peak and Wilson Ridge, enabling trigonometric leveling from three directions using redundant measurements. Thus the accuracy and reliability are both improved.

The property that affects the observational accuracy of trigonometric leveling the most is the determination accuracy of the refraction coefficient k. In the Grove Mountains, surrounded by the snow- and ice-fields of Antarctica, the snow surface primarily reflects sunlight while absorbing little energy. In addition, the wind force is extremely strong during the Antarctic day. Therefore, these elements all tend to make the atmosphere more even and steady, which is not only helpful for the measurement of vertical angles all day long, but also enables the use of a reliable and fundamental theoretical formula that based on the freedom of the atmosphere to calculate refraction coefficients. Therefore, a refraction coefficient with a relatively high accuracy can also be obtained without measuring the temperature gradient. This shows that shortening the distance from the observation site to the target is an efficient way to weaken the effect of atmospheric refraction.

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